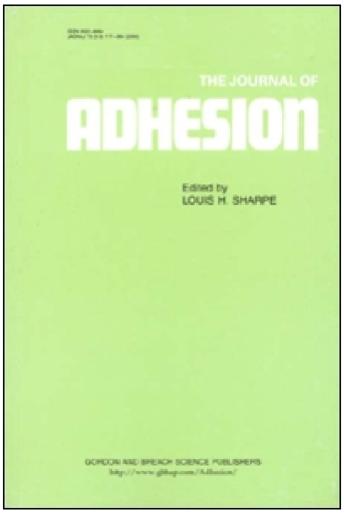

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An Iterative Solution for the Torque Transmission Capability of Adhesively-Bonded Tubular Single Lap Joints with Nonlinear Shear Properties

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The adhesively-bonded tubular single lap joint shows large nonlinear behavior in the load-displacement relationship, because structural adhesives for the joint are usually rubber-toughened, which endows adhesives with nonlinear shear properties. Since the majority of load transfer of the adhesively-bonded tubular single lap joint is accomplished by the nonlinear behavior of the adhesive, its torque transmission capability should be calculated using nonlinear shear properties. However, both the analytic and numerical analyses become complicated if the nonlinear shear properties of the adhesive are included during the calculation of torque transmission capabilities.

In this paper, in order to obtain the torque transmission capabilities easily, an iterative solution which includes the nonlinear shear properties of the adhesive was derived using the analytic solution with the linear shear properties of the adhesive. Since the iterative solution can be obtained very quickly due to its simplicity, it has been found that it can be used in the design of the adhesively-bonded tubular single lap joint.

KEY WORDS adhesive joint; single lap joint; analytic solution; iterative solution; effect of adhesive thickness; nonlinear material behavior of adhesive.

INTRODUCTION

Epoxy adhesives used in joining of mechanical elements are usually rubber filled to increase toughness, which endows adhesives with nonlinear properties. The mechanical elements joined with adhesives with nonlinear properties also behave nonlinearly; therefore, the exact solutions of their behaviors can not be easily obtained. The exact solutions of adhesively-bonded joints were only possible with several assumptions such as linear elastic behaviors of the adhesive and the adherend.

Numerous studies on adhesive joints were performed after Adams[1] obtained the elastic solution of the adhesively-bonded tubular single lap joint. The time-dependent viscoelastic behavior of the adhesive was included in the investigation of the adhesively-bonded joint by Alwar and Nagaraja.[2] The torque transmission capabilities of the partially-tapered tubular scarf joints were investigated by Adams and Peppiatt.[1] The behaviors of the adhesively-bonded joints whose adherends were made of composite materials were investigated by several researchers.[3-5] Several types of the adhesively-bonded joints such as the double lap, the single lap, the scarf, and the stepped-lap joint were analyzed by Hart-Smith.[6] The effect of the adhesive thickness and adherend roughness on the torsional fatigue strength of the adhesively-bonded tubular single lap joint was investigated by Lee *et al.*[7] A failure model for the adhesively-bonded tubular single lap joint was developed by Lee and Lee.[8] The closed-form solution for the torque transmission capability of the adhesively-bonded tubular double lap joint with linear elastic hebaviors of the adhesive and the adherend was derived by Lee and Lee.[9]

Even though there are many such investigations on adhesively-bonded joints, the simple linear elastic solution developed by Adams[1] that underestimates the torque transmission capabilities is usually used in the first stage of design of the adhesively-bonded joint because it is simple and reveals the design parameters.

In this paper, to enhance both the accuracy and speed of the estimation of the torque transmission capability, the linear analytic solution for the adhesively-bonded joint was modified to meet the nonlinear characteristics of the adhesive for joining, and the nonlinear variation of the adhesive shear strain along the adhesive thickness. Then, an iterative method was developed to estimate the stress distributions in the adhesive layer with nonlinear properties. Because the iterative method used a second order differential equation with the nonlinear parameters during calculation, it took a few seconds to obtain the solution. However, more than ten minutes were required to obtain the solution with the finite element method with nonlinear adhesive shear properties.

Due to the very short processing time, the iterative solution was found to be applicable in the design and optimization of the adhesively-bonded tubular single lap joint with nonlinear shear properties as an on-line design tool.

DERIVATION OF THE ITERATIVE SOLUTION

Figure 1 shows the geometric configuration of the adhesively-bonded tubular single lap joint that was analyzed by the iterative method. The iterative solution for the torque

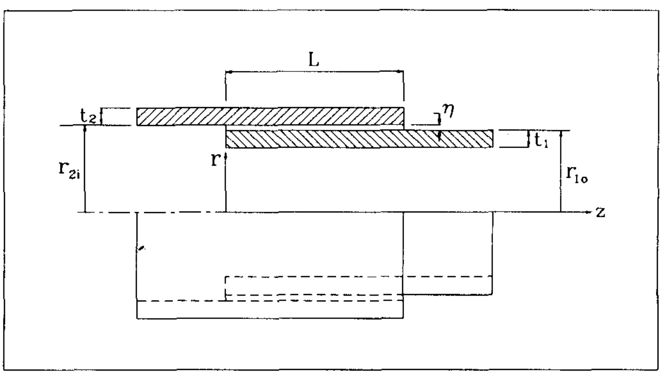


FIGURE 1 Configuration of the adhesively-bonded tubular single lap joint.

transmission capability of the adhesively-bonded tubular single lap joints with non-linear shear properties was based on the analytic solution with linear shear properties. The only difference between the two methods exists in the shear stress-strain relationships of the adhesive.

The first step was based on the formulation of Adams.[1] In this formulation, assumptions that the adherend transfers only the stress component of $\tau_{z\theta}$ and the adhesive transfers only the stress component $\tau_{r\theta}$ were made. The torque, T, that was transmitted by the joint was described by the summation of the torque, T_1, of the adherend 1 and the torque, T_2, of the adherend 2, as follows.

$$T = T_1 + T_2 = \frac{\tau_{1o} J_1}{r_{1o}} + \frac{\tau_{2i} J_2}{r_{2i}} \tag{1}$$

where, r_{1o} and r_{2i} are the outer radius of the adherend 1 and the inner radius of the adherend 2, respectively.

The increments of the torques of adherends 1 and 2 were related to the shear stress, τ_a, of the adhesive at $r = r_{1o}$, as follows.

$$T_2 + dT_2 - T_2 = 2\pi\tau_a r_{1o}^2 dz \tag{2a}$$

$$T_1 + dT_1 - T_1 = -2\pi\tau_a r_{1o}^2 dz \tag{2b}$$

The increments of the torques were calculated at the outer radius of the inner adherend, r_{1o}, because the adhesive stress and strain have the highest values at this point along the adhesive thickness.

By differentiating Equation (2) with respect to z, the following equations were derived.

$$\frac{dT_2}{dz} = 2\pi r_{1o}^2 \tau_a \tag{3a}$$

$$\frac{dT_1}{dz} = -2\pi r_{1o}^2 \tau_a \tag{3b}$$

Figure 2 shows the schematic diagram of the geometric compatibility of the adherends and adhesive. Since the shear strain in the adhesive depends on the radial distance of the adhesive, the difference of the circular displacements of the adherends along the axial distance was calculated by subtracting the integrals of the adhesive shear strains through the adhesive thickness, as follows.

$$\int_{r_{1o}}^{r_{2i}} \gamma_{ar}|_{z+dz}\, dr - \int_{r_{1o}}^{r_{2i}} \gamma_{ar}|_z\, dr = (\gamma_{2i} - \gamma_{1o})\, dz \tag{4}$$

where, γ_{ar}, γ_{1o} and γ_{2i} are the shear strains of the adhesive, adherend 1 and adherend 2, respectively.

In order to simplify the integral form of Equation (4), a nondimensional variable, ω, which has the role of a weighting factor was introduced, using the shear strain of the adhesive at the inner interface, γ_a, and the adhesive thickness, η, as follows. At $z = z$

$$\int_{r_{1o}}^{r_{2i}} \gamma_{ar}|_z\, dr = \omega \cdot \gamma_a \cdot \eta \tag{5}$$

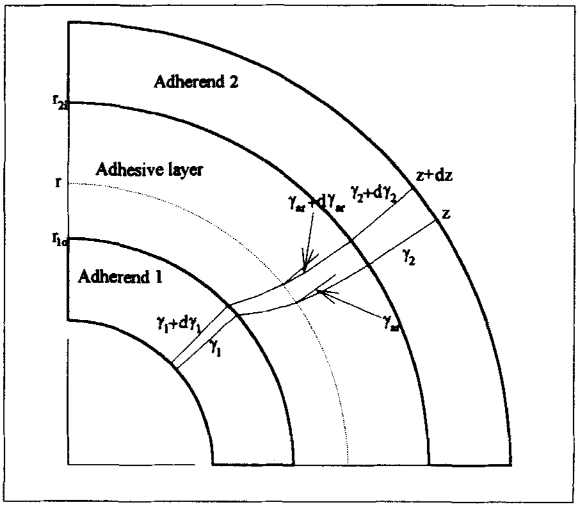


FIGURE 2 Geometric compatibility of the adherends and adhesive.

Then, at $z = z + dz$

$$\int_{r_{1o}}^{r_{2i}} \gamma_{ar}|_{z+dz}\, dr = (\omega + d\omega)\cdot(\gamma_a + d\gamma_a)\cdot\eta$$

$$= (\omega\cdot\gamma_a + \omega\cdot d\gamma_a + d\omega\cdot\gamma_a + d\omega\cdot d\gamma_a)\cdot\eta$$

$$\approx (\omega\cdot\gamma_a + \omega\cdot d\gamma_a + d\omega\cdot\gamma_a)\cdot\eta \tag{6}$$

Substituting Equations (5) and (6) into Equation (4), the following equation was obtained.

$$\frac{d\gamma_a}{dz} \approx \frac{\gamma_{2i} - \gamma_{1o}}{\omega\cdot\eta} - \frac{d\omega}{dz}\cdot\frac{\gamma_a}{\omega} \tag{7}$$

Since the adhesive used in joining operations usually has highly nonlinear shear properties due to rubber toughening, in this paper, the shear stress of the adhesive was represented by the function of the shear strain of the adhesive, as follows.

$$\tau_a = f(\gamma_a) \tag{8}$$

Then, the increment of the shear stress in the adhesive was expressed as given below.

$$d\tau_a = \frac{df(\gamma_a)}{d\gamma_a}\cdot d\gamma_a \tag{9}$$

Differentiating Equation (3a) with respect to z and using Equations (7) and (9), the following Equation was derived.

$$\frac{d^2 T_2}{dz^2} = 2\pi r_{1o}^2 \frac{df(\gamma_a)}{d\gamma_a}\left(\frac{\gamma_{2i} - \gamma_{1o}}{\omega\eta} - \frac{d\omega}{dz}\frac{\gamma_a}{\omega}\right) \tag{10}$$

Eliminating T_2, γ_{2i} and γ_{1o} using τ_{2i} and τ_{1o}, Equation (10) can be modified as shown.

$$\frac{J_2}{r_{2i}}\frac{d^2\tau_{2i}}{dz^2}=2\pi r_{1o}^2\frac{df(\gamma_a)}{d\gamma_a}\frac{1}{\omega}\left\{\frac{1}{\eta}\left(\frac{\tau_{2i}}{G_2}-\frac{\tau_{1o}}{G_1}\right)-\frac{d\omega}{dz}\gamma_a\right\} \tag{11}$$

From Equations (1), τ_{1o} was represented by,

$$\tau_{1o}=\frac{r_{1o}}{J_1}\left(T-\frac{\tau_{2i}J_2}{r_{2i}}\right) \tag{12}$$

Substituting Equation (12) into Equation (11), the second-order ordinary differential equation for the shear stress distribution of the adherend 2 was constructed incorporating the nonlinear shear properties of the adhesive, as follows.

$$\frac{J_2}{r_{2i}}\frac{d^2\tau_{2i}}{dz^2}=2\pi r_{1o}^2\frac{df(\gamma_a)}{d\gamma_a}\frac{1}{\omega}\left\{\frac{1}{\eta}\left(\frac{1}{G_2}+\frac{r_{1o}J_2}{r_{2i}J_1G_1}\right)\tau_{2i}-\left(\frac{r_{1o}T}{\eta G_1J_1}+\frac{d\omega}{dz}\gamma_a\right)\right\} \tag{13}$$

Equation (13) was written using two parameters a_1 and a_2 as shown below.

$$\frac{d^2\tau_{2i}}{dz^2}+\alpha_1(\gamma_a)\tau_{2i}+\alpha_2(\gamma_a)=0 \quad (14$$

where;

$$\alpha_1=-\frac{2\pi r_{1o}^2r_{2i}}{\omega\eta J_2}\frac{df(\gamma_a)}{d\gamma_a}\left(\frac{1}{G_2}+\frac{r_{1o}J_2}{r_{2i}J_1G_1}\right)$$

$$\alpha_2=\frac{2\pi r_{1o}^2r_{2i}}{J_2}\frac{df(\gamma_a)}{d\gamma_a}\left(\frac{r_{1o}T}{\eta G_1J_1}+\frac{d\omega}{dz}\gamma_a\right)$$

Equation (14) was rewritten in the iterative form,

$$\frac{d^2\tau_{2i}^{(i+1)}}{dz^2}+\alpha_1^{(i)}(\gamma_a^{(i)})\tau_{2i}^{(i+1)}+\alpha_2^{(i)}(\gamma_a^{(i)})=0 \tag{15}$$

The boundary conditions at both edges were,

$$\begin{aligned}\tau_{2i}&=0 &&\text{at } z=0\\ \tau_{2i}&=\frac{T\cdot r_{2i}}{J_2} &&\text{at } z=L\end{aligned} \tag{16}$$

The parameters $\alpha_1^{(i)}(\gamma_a^{(i)})$ and $\alpha_2^{(i)}(\gamma_a^{(i)})$ in Equation (15) represent the functions of the strains of the adhesive during the i-th iteration. To obtain the values of the parameters of the current step, the strains of the adhesive of the previous step were used. After obtaining the stress distributions at the inner surface of the adherend 2 from Equation (15), the stress distributions of the outer surface of the adherend 1 were calculated using Equation (12).

The shear stress distributions of the adhesive were calculated using Equation (3a):

$$\tau_a=\frac{1}{2\pi r_{1o}^2}\frac{J_2}{r_{2i}}\frac{d\tau_{2i}}{dz} \tag{17}$$

Since the torque is constant along the adhesve thickness, the weighting factor, ω, was calculated by Equation (5):

$$\omega = \frac{\int_{r_{1o}}^{r_{2i}} \gamma_{ar}\, dr}{\gamma_a \cdot \eta} = \frac{\int_{r_{1o}}^{r_{2i}} f^{-1}\left(\frac{r_{1o}^2}{r^2}\tau_a\right) dr}{\gamma_a \cdot \eta} \tag{18}$$

The weighting factor can be calculated using Equation (18) only when the shear stress-strain relation is described by a functional equation. Therefore, the iterative method requires the functional form of the stress-strain relation.

To improve the convergence of the iterative solution, the feedback variable of Equation (15), $\gamma_a^{(i)}$ was modified to $\gamma_a^{(i)}$ using the following Equation.

$$\gamma_a^{(i*)} = \theta \cdot \gamma_a^{(i)} + (1 - \theta) \cdot \gamma_a^{(i-1)} \tag{19}$$

where the constant θ has a value between 0 and 1.

During the calculation of the torque transmission capability, it was found the $\theta = 0.5$ was satisfactory for the convergence of the solution.

EXAMPLES

In order to investigate the applicability of the iterative solution, the torque transmission capability of the adhesively-bonded tubular single lap joints with nonlinear shear properties was calculated using the iterative solution.

The nonlinear shear properties of rubber-toughened adhesives were modeled by the following exponential form, as an example.

$$\tau_a = f(\gamma_a) = \tau_{af} \cdot \left\{1 - \exp\left(-\frac{G_a}{\tau_{af}} \cdot \gamma_a\right)\right\} \tag{20}$$

where τ_{af} and G_a represent the ultimate shear strength and shear modulus of adhesive, respectively.

In this example, the ultimate shear strength of 30 MPa and shear modulus of 461 MPa, and the ultimate shear strain, γ_{af}, of 0.4 were selected. These values are typical of rubber-toughened epoxy adhesives. Figure 3 shows two shear stress-strain curves: Equation (20) for the iterative solution and the experimental result. In Figure 3, Equation (20) matched well with the experimental result for IPCO 9923, a rubber-toughened epoxy adhesive manufactured by the Imperial Polychemicals Corporation (Azusa, California, USA). The shear stress-strain relation was obtained from a torsion test with tubular specimens. Table I shows the dimensions of the adhesively-bonded joint used for the analysis.

In order to show the effect of nonlinear shear properties on the shear stress and strain distributions in the adhesive layer, it was assumed that the adherends were not failed. Through the numerical calculations with the iterative method, it was found that the stress and strain distributions, as well as the torque transmission capabilities, of the adhesively-bonded tubular single lap joints with nonlinear shear properties were obtained within a few seconds. The iterative solution was found to be very efficient,

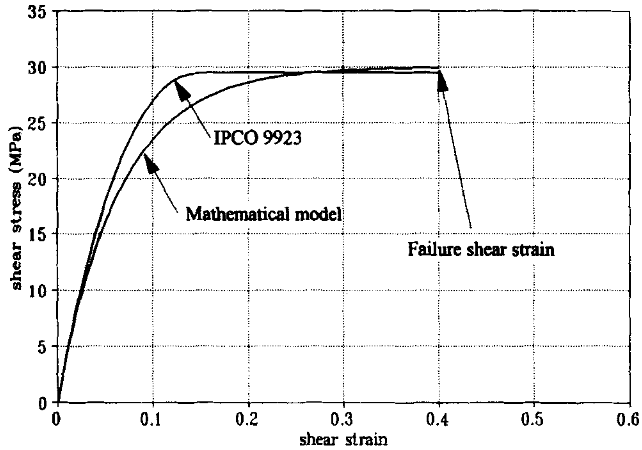


FIGURE 3 Shear stress-strain curve of the typical rubber-toughened adhesive.

TABLE I
The dimensions and some adhesive properties of the adhesively-bonded joint

r_{2i}(mm)	15.0
t_2(mm)	3.0
r_{1o}(mm)	$r_{2i} - \eta$
t_1(mm)	3.0
L(mm)	20.0
η(mm)	0.1, 0.5, 1.0
G_1, G_2(GPa)	80.8
G_a	described in Equation (20)

because it took more than ten minutes to obtain the solution by a two-dimensional finite element method with 8-node, axisymmetric-isoparametric elements even though relatively coarse meshes were employed in the axial direction.

Figure 4 shows the weighting factor, ω, that depends on the joint size and shear stress of the adhesive at the inner interface, which means that the nonlinearity of the shear strain in the adhesive exists. The weighting factor decreased rapidly as the shear stress of the adhesive approached the shear strength of the adhesive and decreased also as the adhesive thickness increased, at the same shear stress.

Figure 5 shows the torque transmission capabilities of the adhesively-bonded tubular single lap joint with respect to the maximum shear strain of the adhesive with 0.1 mm and 0.2 mm adhesive thickness. These were calculated both by the iterative method and the finite element method with axisymmetry. The trend of the torque transmission capability is similar to the shear stress-strain curve of Figure 3. The iterative method yielded the torque within 5% error compared with that obtained by the finite element method at the maximum shear strain.

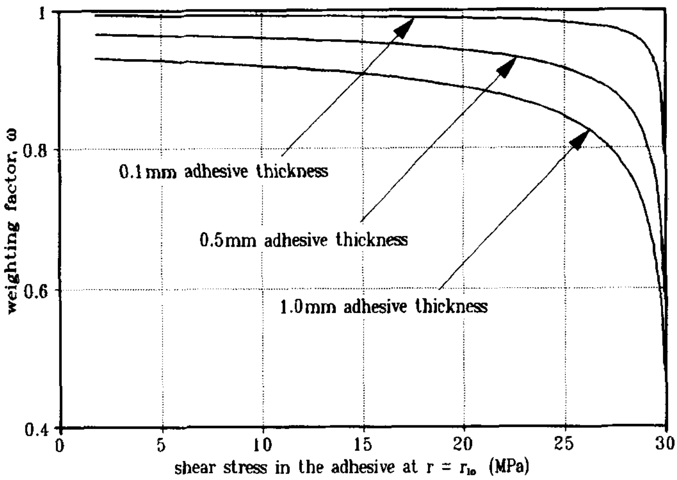


FIGURE 4 Weighting factor, ω, w.r.t. the shear stress in the adhesive at $r = r_{1o}$ (adhesive thickness = 0.1, 0.5, 1.0 mm).

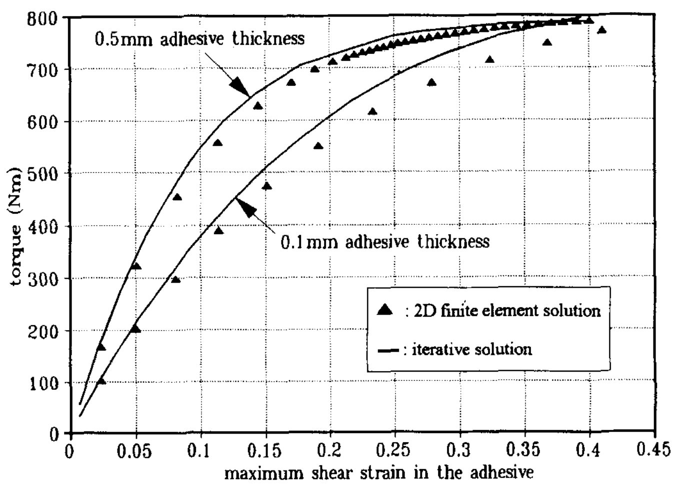
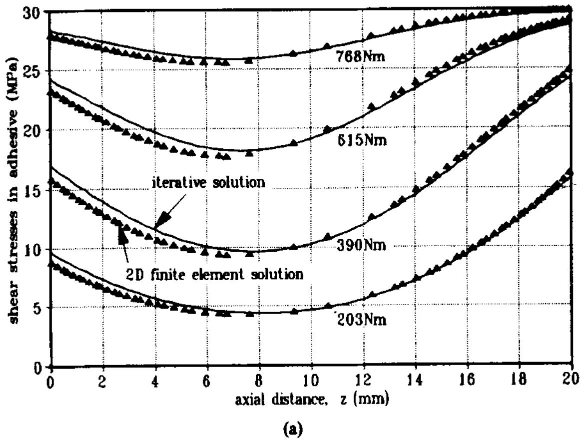


FIGURE 5 Torque transmission capabilities of the adhesively-bonded tubular single lap joint w.r.t. the maximum shear strain of the adhesive (adhesive thickness = 0.1 mm, 0.5 mm).

Figure 6 shows the shear stress distributions in the adhesive of the joint with 0.1 mm and 0.5 mm adhesive thickness that were calculated by the iterative method and the finite element method, with respect to the applied torque transmission capability. The stress relaxation due to the material nonlinearity of the adhesive was observed at the ends of the lap when the shear stress of the adhesive was high.



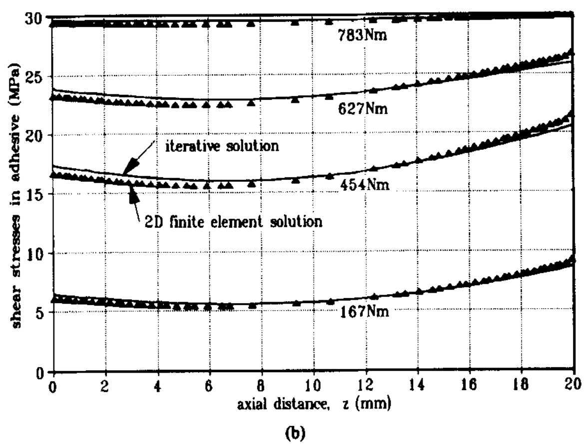


FIGURE 6 Shear stress distribution in the adhesive w.r.t. the applied torque levels. (a) adhesive thickness = 0.1 mm, (b) adhesive thickness = 0.5 mm.

Figure 7 shows the torque transmission capability of teh adhesively-bonded tubular single lap joint with respect to the thickness of the adherend 2 when the thickness of the adherend 1 was fixed at 3.0 mm. The maximum torque was observed when the thickness of adherend 2 was 1.8 mm, which makes the polar moments of inertia of the adherend 1 and adherend 2 equal.

Figure 8 shows the torque transmission capability of the adhesively-bonded tubular single lap joint with respect to the adhesive bonding length when the thicknesses of adherend 2 were 1.8 mm and 3.0 mm andthe adhesive thickness was 0.1 mm. As the bonding length increased up to 25 mm, the torque transmission capability increased, then saturated with the torque transmission capabilities of 1008 N·m and 881 N·m when the thicknesses of the adherend 2 were 1.8 mm and 3.0 mm, respectively.

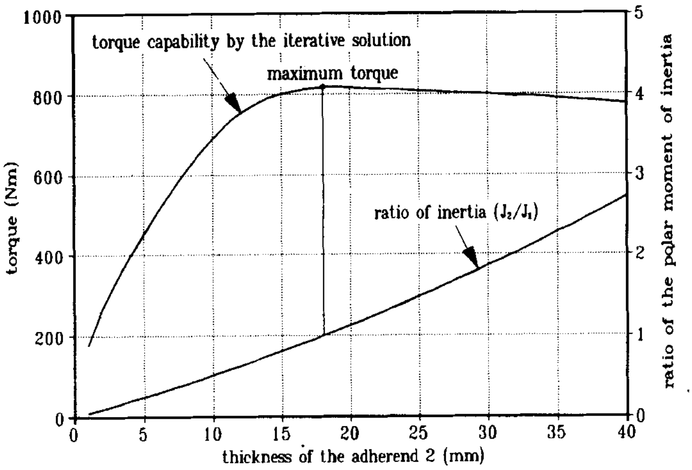


FIGURE 7 Torque capability and ratio of the polar moment of inertia of the adherends w.r.t. the thickness of the adherend 2 when the thickness of the adherend 1 is 3.0 mm (adhesive thickness = 0.1 mm).

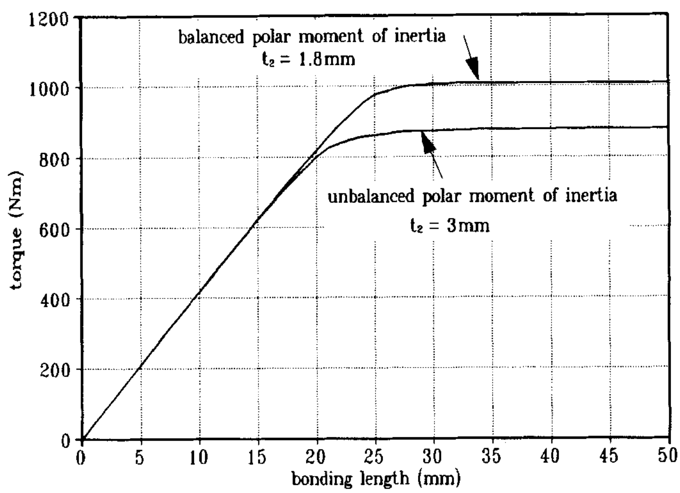


FIGURE 8 Torque capabilities of the balanced joint and the unbalanced joint w.r.t. the bonding length (adhesive thickness = 0.1 mm).

CONCLUSIONS

The iterative solution for the torque transmission capability of the adhesively-bonded tubular single lap joint with nonlinear shear properties of adhesive was derived using the solution with the linear shear properties of adhesives. The variation of the shear stress and strain in the adhesive along the adhesive thickness were taken into consideration in the derivation.

Due to the simplicity of the iterative solution, the torque transmission capability was obtained with very short computational time and it was found that the method could be used in the design of the adhesively-bonded joint with nonlinear shear properties of adhesives as an on-line design tool.

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